Effective dissipative dynamics for polarized photons

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In the framework of open quantum systems, the propagation of polarized photons can be effectively described using quantum dynamical semigroups. These extended time evolutions induce irreversibility and dissipation. Planned, high sensitive experiments, both in the laboratory and in space, will be able to put stringent bounds on these nonstandard effects.

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The propagation of polarized photons in an optical active media is a theoretically well understood physical process; it can be effectively modeled by means of linear transformations acting in the space of polarization states [1–4]. In the presence of dissipation however, a more general formalism in terms of density matrices is usually needed: it can be physically motivated in the framework of quantum open systems [5–8].

These systems can be thought of as being subsystems in interaction with large environments. Although the time evolution of the total system follows the standard quantum mechanical rules, the dynamics of the subsystem, obtained by the elimination of the environment degrees of freedom, is usually very involved, developing dissipation and irreversibility. In many physical instances (essentially when the interaction between the subsystem and the environment can be considered to be weak) the subdynamics can be explicitly described in terms of quantum dynamical semigroups, i.e., by mean of families of linear maps, transforming density matrices into density matrices, and satisfying the conditions of entropy increase, forward in time composition and complete positivity [5–7].

Various physical situations encountered in quantum optics can be studied within this framework [8–10], and indeed, many developments in the theory of quantum dynamical semigroups have been motivated through these applications. However, the formalism is very general and has also been applied to describe many other physical systems, ranging from statistical models [5–7], to molecular systems [11], to the interaction of a microsystem with a measuring apparatus [12–14].

Further, quantum dynamical semigroups have been recently applied to the study of dissipation and irreversibility in elementary particle phenomena [14–16]. The original physical motivation for these investigations come from quantum gravity: due to the quantum fluctuation of the gravitational field and the presence of virtual black holes, space-time should loose its ''continuum'' aspect at distances of the order of Planck's scale and assume a ''foam'' like behavior [17]. As a consequence, new, nonstandard phenomena can arise, leading to loss of quantum coherence [17–23].

Recent studies based on the dynamics of extended objects (strings and branes) also support from a more fundamental point of view this possibility [24,25]. Unfortunately, our present knowledge of string dynamics does not allow to quantify precisely the magnitude of the induced nonstandard, dissipative phenomena. In any case, they should produce very small effects; these are in fact suppressed by at least one inverse power of the Planck mass, as a rough dimensional analysis suggests.

Nevertheless, for particular physical systems, involving interference phenomena, these dissipative effects might be in the reach of present and future experiments. Indeed, detailed investigations involving neutral meson systems, neutron interferometry and neutrino oscillations using quantum dynamical semigroups have already allowed to derive order of magnitude limits on some of the phenomenological constants parametrizing the new phenomena using current experimental data [26–29]. More detailed results are expected when new data, in particular involving correlated neutral mesons, will be available [30].

Photon interferometry, and more in general optical physics, is surely another obvious instance in which nonstandard, dissipative effects induced by fundamental dynamics at Planck's scale might be relevant. Many interferometriclike experiments have been devised (and will be operational in the near future) for the study of a wide range of different phenomena, from the analysis of laser physics, to tests of quantum mechanics, from the detection of gravitational waves, to the study of astrophysical and cosmological objects; therefore, it appears relevant to discuss in detail to what extent dissipation can affect all those observations.

We shall concentrate our attention on the physics of polarized photons, since it seems to offer many experimental opportunities for detecting the new, dissipative phenomena. Quite in general, these effects can be parametrized in terms of six phenomenological constants, whose presence modify the behavior of various physical observables. Explicit expressions for some of these observables will be given using useful approximations; they can be of help in fitting experimental data. Indeed, as discussed at the end, planned, future experimental setups, both in the laboratory and in space, should reach the required sensitivity to measure with good accuracy at least some of the dissipative parameters. This is surely an important motivation for further, more detailed investigations.

Polarized photons can be effectively described by means of a two-dimensional Hilbert space, the space of helicity states [3,4]. Any vector in this space represents a given polarization and can be identified by two angles θ and φ:

$$|\theta,\varphi\rangle = \cos\theta|+\rangle + e^{i\varphi}\sin\theta|-\rangle, \tag{1}$$

where $|+\rangle$ and $|-\rangle$ are two orthonormal basis vectors, representing linearly polarized states. Another convenient basis in this space is given by the circularly polarized states:

$$|R\rangle = \frac{1}{\sqrt{2}}(|+\rangle + i|-\rangle),$$

$$|L\rangle = \frac{1}{\sqrt{2}}(|+\rangle - i|-\rangle). \tag{2}$$

With respect to this basis, any (partially) polarized photon state can be represented by a 2×2 density matrix ρ; this is a hermitian, positive operator, i.e., with positive eigenvalues, and constant trace. In particular, to the state (1) there corresponds the following matrix:

$$\rho_{\theta,\varphi} = \frac{1}{2}\begin{bmatrix} 1+\sin\varphi\sin 2\theta & \cos 2\theta - i\cos\varphi\sin 2\theta \\ \cos 2\theta + i\cos\varphi\sin 2\theta & 1-\sin\varphi\sin 2\theta \end{bmatrix}; \tag{3}$$

representing a pure state, this matrix is a projector: $(\rho_{\theta,\varphi})^2 = \rho_{\theta,\varphi}$.

Quite in general, the dynamics of any state ρ can be described by an equation of the following form:

$$\frac{\partial\rho(t)}{\partial t} = -iH\rho(t) + i\rho(t)H + \mathcal{L}[\rho(t)], \tag{4}$$

where H is the standard Hamiltonian, and $\mathcal{L}$ is a linear map that can be written as

$$\mathcal{L}[\rho] = -\frac{1}{2}\sum_j (L_j^\dagger L_j\rho + \rho L_j^\dagger L_j) + \sum_j L_j\rho L_j^\dagger, \tag{5}$$

while the L's form a suitable collection of operators, such that $\Sigma_j L_j^\dagger L_j$ is a well-defined 2×2 matrix. Indeed, one can show that this equation is the (unique) result of very basic physical requirements that the complete time-evolution, $\tau_t:\rho(0)\mapsto\rho(t)$ needs to satisfy; the one-parameter (=time) family of linear maps τ_t should transform density matrices into density matrices and have the property of increasing the (von Neumann) entropy, $S=-\mathrm{Tr}[\rho(t)\ln\rho(t)]$, of obeying the semigroup composition law, $\tau_t[\rho(t')]=\rho(t+t')$, for $t,t'\geqslant 0$, of being completely positive. These are the characteristic properties of quantum dynamical semigroups, that are therefore generated by equations of the form (4), with Eq. (5) [5–7].

As mentioned in the introductory remarks, quantum dynamical semigroups have been used to describe a wide range of phenomena related to the study of open quantum systems; in particular, they have been applied to analyze nonstandard, dissipative effects in the propagation and decay of neutral meson systems. Although the basic general motivation behind these treatments is that quantum phenomena at a fundamental scale produce loss of phase coherence, one should always keep in mind that the form (4), (5) of the evolution equation is very general and independent from the actual microscopic dynamical mechanism responsible for the dissipative effects: in view of the properties they satisfy, any physically sensible description of decoherence phenomena must be based on Eqs. (4), (5). For these reasons, the discussion in the following, although applied to a specific model, retains its validity in a much more general framework.

In connection with these observations, one should add a further general comment on the time evolution $\rho(t)$. In view of the interpretation of its eigenvalues as probabilities, the density matrix $\rho(t)$ needs to be a positive operator for all times; this is clearly a crucial requirement for the consistency of the whole formalism, and it is satisfied in all situations only if the map $\rho(0)\mapsto\rho(t)$ is completely positive. Roughly speaking, this amounts to the requirement of positivity for the density matrix of a larger system, involving the coupling with an extra, auxiliary finite-dimensional system (for details, see [5–7,15,25]). This property is trivially satisfied by ordinary (unitary) time evolutions, and turns out to be crucial in properly treating effects of irreversibility in correlated systems [31]. For this reason, in order to study possible nonstandard, dissipative effect even in simpler, uncorrelated systems the phenomenological equations (4), (5) should always be used.

In the chosen basis (2), the hamiltonian H has generically the form

$$H = \begin{bmatrix} \omega_0+\omega_3 & \omega_1 - i\omega_2 \\ \omega_1 + i\omega_2 & \omega_0-\omega_3 \end{bmatrix}, \tag{6}$$

where ω_0 is the average photon energy, while the real parameters $\omega_1,\omega_2,\omega_3$ produce the level splitting $\omega\equiv(\omega_1^2+\omega_2^2+\omega_3^2)^{1/2}$. In the following, we have kept ω nonvanishing in order to take into account possible propagation in an optical active media. Although the most natural way of realizing this is through the use of a suitable crystal ([3,4], see also [32,33]), many other unconventional mechanisms leading to birefringence effects have been discussed in the literature. They involve the action of external fields [34], Chern-Simons [35], or more in general Lorentz and *CPT*-violating modifications of Maxwell Lagrangian [36], extensions of general relativity [37], and quantum gravity phenomena [38].[1] Furthermore, one should take into account that in gen-

[1]Other unconventional Planck's scale phenomena affecting photon propagation, but not directly leading to birefringence, have also been studied; see [39] and references therein.

eral the formalism of open quantum systems can also lead to nonvanishing Hamiltonian contributions, so that, even in absence of other physical mechanisms, birefringence effects should always be present as the result of the interaction with the environment [5,25].

On the other hand, the additional piece $\mathcal{L}[\rho]$ in Eq. (5) induces a mixing-enhancing mechanism, leading to irreversibility and possible loss of quantum coherence. Being a linear map, it can be represented as a 4×4 matrix acting on the entries of ρ. It can be fully parametrized in terms of six real phenomenological constants, a, b, c, α, β, and γ, satisfying the following inequalities:

$$2R\equiv\alpha+\gamma-a\geqslant0,\quad RS-b^2\geqslant0,$$

$$2S\equiv a+\gamma-\alpha\geqslant0,\quad RT-c^2\geqslant0,$$

$$2T\equiv a+\alpha-\gamma\geqslant0,\quad ST-\beta^2\geqslant0,$$

$$RST-2bc\beta-R\beta^2-Sc^2-Tb^2\geqslant0, \tag{7}$$

direct consequence of the property of complete positivity [5,15]. A convenient explicit expression for the right-hand side (RHS) of Eq. (4) can be obtained by decomposing the 2×2 density matrix ρ in terms of the Pauli matrices σ_i, $i=1,2,3$, and the identity σ_0:

$$\rho=\frac{1}{2}\sum_{\mu=0}^{3}\rho_\mu\sigma_\mu. \tag{8}$$

One can then rewrite the evolution (4) as a Schrödinger-like equation for the abstract vector $|\rho(t)\rangle$ of components $(\rho_0,\rho_1,\rho_2,\rho_3)$:

$$\frac{\partial}{\partial t}|\rho(t)\rangle=-2\mathcal{K}|\rho(t)\rangle. \tag{9}$$

The 4×4 matrix $\mathcal{K}$ includes both the Hamiltonian piece, $-i[H,\rho]$, and the contribution $\mathcal{L}[\rho]$, and takes the block-diagonal form

$$[\mathcal{K}_{\mu\nu}]=\begin{bmatrix}0 & 0\\ 0 & \mathcal{H}_{ij}\end{bmatrix},$$

$$[\mathcal{H}_{ij}]=\begin{bmatrix}a & b+\omega_3 & c-\omega_2\\ b-\omega_3 & \alpha & \beta+\omega_1\\ c+\omega_2 & \beta-\omega_1 & \gamma\end{bmatrix},\quad i,j=1,2,3. \tag{10}$$

The formal solution of Eq. (9) involves the exponentiation of $\mathcal{K}$:

$$|\rho(t)\rangle=\mathcal{M}(t)|\rho(0)\rangle,\quad [\mathcal{M}_{\mu\nu}(t)]=\begin{bmatrix}1 & 0\\ 0 & \mathcal{N}_{ij}(t)\end{bmatrix},$$

$$\mathcal{N}(t)=e^{-2\mathcal{H}t}. \tag{11}$$

Notice that the time evolution generated by Eqs. (4), (5) is trace-preserving; therefore, from the initial normalization condition $\mathrm{Tr}[\rho(0)]=1$, one immediately deduces that $\rho_0(t)=1$ for all times, as it is apparent from Eq. (11).

Any physical property of a polarized photon beam can be extracted from the density matrix $\rho(t)$ by computing its trace with suitable hermitian operators. In particular, the observables that correspond to the Pauli matrices σ_i and the identity σ_0 give the so-called (normalized) Stokes polarization parameters. From the decomposition (8), it is then clear that the vector $|\rho\rangle$ precisely represents a normalized Stokes vector; therefore, the 4×4 real matrix $\mathcal{M}(t)$ in Eq. (11) corresponds to the Mueller matrix connecting the initial Stokes vector $|\rho(0)\rangle$ with the evolved one $|\rho(t)\rangle$ at time t [1–4].

For instance, in absence of the additional piece $\mathcal{L}[\rho]$, the matrix $\mathcal{N}(t)$ in Eq. (11) can be explicitly written as

$$\mathcal{N}_{ij}(t)=\delta_{ij}-\frac{\sin 2\omega t}{\omega}\mathcal{H}_{ij}+\frac{2\sin^2\omega t}{\omega^2}\mathcal{H}_{ij}^2,\quad i,j=1,2,3. \tag{12}$$

The physical meaning of the corresponding Mueller matrix $\mathcal{M}(t)$ can be most simply obtained by taking $\omega_1=\omega_2=0$, or alternatively by switching to the basis in which the Hamiltonian H is diagonal. In this case $\mathcal{M}(t)$ becomes block-diagonal:

$$\mathcal{M}(t)=\begin{bmatrix}1 & & \\ & \mathcal{R}(t) & \\ & & 1\end{bmatrix},\quad \mathcal{R}(t)=\begin{bmatrix}\cos 2\omega t & -\sin 2\omega t\\ \sin 2\omega t & \cos 2\omega t\end{bmatrix} \tag{13}$$

it represents a rotator: in fact, for linearly polarized states,

$$|\rho(0)\rangle=\begin{pmatrix}1\\ \cos 2\theta_0\\ \sin 2\theta_0\\ 0\end{pmatrix}, \tag{14}$$

the direction of polarization, initially along θ_0, is rotated by an angle ωt, proportional to the elapsed time.

More in general, any observable $\mathcal{O}$ can be decomposed as in Eq. (8),

$$\mathcal{O}=\sum_{\mu=0}^{3}\mathcal{O}_\mu\sigma_\mu, \tag{15}$$

so that its corresponding mean value is given by

$$\langle\mathcal{O}(t)\rangle\equiv\mathrm{Tr}[\mathcal{O}\rho(t)]=\sum_{\mu=0}^{3}\mathcal{O}_\mu\rho_\mu(t). \tag{16}$$

Of particular interest is the observable that correspond to the fully polarized state in Eq. (3); the probability that the evolved vector $|\rho(t)\rangle$ be in such a state is then given by

$$\mathcal{P}_{\theta,\varphi}(t)=\langle\rho_{\theta,\varphi}\rangle=\frac{1}{2}[1+\rho_1(t)\cos 2\theta+\rho_2(t)\cos\varphi\sin 2\theta+\rho_3(t)\sin\varphi\sin 2\theta]. \tag{17}$$

The corresponding intensity curve that this probability produces can be compared directly with the experiment, provided explicit expressions for the entries of the matrix $\mathcal{M}(t)$ in Eq. (11) are given.

Formally, this can be obtained by studying the eigenvalue problem for the 3×3 matrix $\mathcal{H}$ in Eq. (10):

$$\mathcal{H}|v^{(k)}\rangle=\lambda^{(k)}|v^{(k)}\rangle,\quad k=1,2,3. \tag{18}$$

The three eigenvalues $\lambda^{(1)},\lambda^{(2)},\lambda^{(3)}$ satisfy the cubic equation:

$$\lambda^3+r\lambda^2+s\lambda+w=0, \tag{19}$$

with real coefficients,

$$r\equiv-(\lambda^{(1)}+\lambda^{(2)}+\lambda^{(3)})=-(a+\alpha+\gamma), \tag{20a}$$

$$s\equiv\lambda^{(1)}\lambda^{(2)}+\lambda^{(1)}\lambda^{(3)}+\lambda^{(2)}\lambda^{(3)}=a\alpha+a\gamma+\alpha\gamma-b^2-c^2-\beta^2+\omega^2, \tag{20b}$$

$$w\equiv-\lambda^{(1)}\lambda^{(2)}\lambda^{(3)}=a(\beta^2-\omega_1^2)+\alpha(c^2-\omega_2^2)+\gamma(b^2-\omega_3^2)-a\alpha\gamma-2bc\beta-2b\omega_1\omega_2-2c\omega_1\omega_3-2\beta\omega_2\omega_3. \tag{20c}$$

Via Cardano's formula [40], the corresponding solutions can be expressed in terms of the associated discriminant $\mathcal{D}=p^3+q^2$, $p=s/3-(r/3)^2$, $q=(r/3)^3-rs/6+w/2$; the eigenvalues are either all real $(\mathcal{D}\leqslant 0)$, or one is real and the remaining two are complex conjugate $(\mathcal{D}>0)$. The degenerate case $\mathcal{D}=0$ occurs when two real eigenvalues are equal; all three coincide for $p=q=0$.

Then, using the fact that the matrix $\mathcal{H}$ itself obeys Eq. (19), one can show that the entries of $\mathcal{N}$ in Eq. (11) can be written as

$$\mathcal{N}_{ij}(t)=\sum_{k=1}^{3}e^{-2\lambda^{(k)}t}\left[\frac{([\lambda^{(k)}]^2+r\lambda^{(k)}+s)\delta_{ij}+(\lambda^{(k)}+r)\mathcal{H}_{ij}+\mathcal{H}_{ij}^2}{3[\lambda^{(k)}]^2+2r\lambda^{(k)}+s}\right],\quad i,j=1,2,3. \tag{21}$$

[The expression in Eq. (12) is just a particularly simple example of this general formula.]

When $\omega=0$, i.e., $\omega_1=\omega_2=\omega_3=0$, the matrix $\mathcal{H}$ is real, symmetric and non-negative, as guaranteed by the inequalities (7); therefore, its eigenvalues are all real and non-negative: $\mathcal{D}<0$ and $p<0$. Only for sufficiently large values of ω_1, ω_2 or ω_3 the discriminant $\mathcal{D}$ becomes positive and complex eigenvalues may appear; note that their real part are always non-negative, since in general the quantum dynamical semigroup generated by Eqs. (4), (5) is bounded for any t [41]. Therefore, the general behavior of the Mueller matrix $\mathcal{M}(t)$ in Eq. (11) depends on the relative magnitude of the constants ω_1, ω_2 and ω_3 with respect to the dissipative parameters a, b, c, α, β and γ; only when the latter are small compared to the former an oscillatory behavior is possible, while exponential dumping terms prevail, when dissipation is the dominant phenomena.

In particular, when $\det(\mathcal{H})\equiv-w\neq0$, one can show that, in presence of dissipation, the real parts of the three eigenvalues $\lambda^{(1)}$, $\lambda^{(2)}$, $\lambda^{(3)}$ are all strictly positive [29]. Then, for large times, the dumping terms dominate and the Mueller matrix $\mathcal{M}(t)$ becomes that of a total depolarizer:

$$\mathcal{M}(t)\underset{t\to\infty}{\sim}\mathrm{diag}(1,0,0,0). \tag{22}$$

Indeed, in this case the degree of polarization, defined by

$$\Pi(t)=(1-\det[\rho(t)])^{1/2}, \tag{23}$$

asymptotically vanishes, independently from the initial state $\rho(0)$.

The situation is more complicated in presence of zero eigenvalues, since the additional cubic condition $w=0$ needs to be imposed. Looking at the explicit expression in Eq. (20c) and recalling the inequalities (7), it is clear that a vanishing w can be obtained only for very special values of the parameters ω_1, ω_2, ω_3 and a, b, c, α, β, γ. As a simplifying assumption, let us take $\omega_1=\omega_2=0$; keeping ω_3 arbitrary, the only way to set $w=0$ is then to impose $\gamma=0$; indeed, the inequalities (7) further imply $b=c=\beta=0$ and $a=\alpha$. Most of the entries of $\mathcal{N}(t)$ are still exponentially suppressed for large t; however, the presence of the zero eigenvalues now implies $\mathcal{N}_{33}(t)=1$, so that the asymptotic form of the Mueller matrix becomes

$$\mathcal{M}(t)\underset{t\to\infty}{\sim}\mathrm{diag}(1,0,0,1). \tag{24}$$

In this case, the degree of polarization (23) vanishes only for states that initially are linearly polarized, as in Eq. (14).

The large-time behaviors in Eqs. (22) and (24) [for the special case $\omega_1=\omega_2=\gamma=0$] are characteristic of the presence of the dissipative contribution (5) to the evolution equation (4) and can be put to experimental tests. However, for the determination of the parameters a, b, c, α, β, and γ, explicit expressions for the elements of the matrix $\mathcal{N}(t)$, for finite t, are needed. These are in general very complicated [cf. Eq. (21)]; nevertheless, having in mind possible comparisons with experimental data, their study in suitable approximations seems appropriate.

In order to simplify the treatment, we shall henceforth adopt the eigenstates of the Hamiltonian in Eq. (6) as basis states; notice that the physical observables, e.g., the probability in Eq. (17), being the result of a trace operation, are independent from any choice of basis. [Alternatively, one can assume $\omega_1=\omega_2=0$.]

When the magnitude of the dissipative, nonstandard parameters is large or comparable with respect to that of ω, a useful working assumption is to take c and β to be much smaller than the remaining constants: indeed, this choice is perfectly compatible with the constraints of complete positivity in Eq. (7). To lowest order, the matrix $\mathcal{H}$ becomes block diagonal and a manageable expression for the entries of the Mueller matrix $\mathcal{M}(t)$ can be obtained. Explicitly, $\mathcal{M}(t)$ can be written as the product

$$\mathcal{M}(t)=\mathcal{M}_D(t)\cdot\mathcal{M}_R(t), \tag{25}$$

where $\mathcal{M}_D(t)$ is diagonal and contains exponential dumping factors

$$\mathcal{M}_D(t)=\mathrm{diag}(1,e^{-At},e^{-At},e^{-2\gamma t}), \tag{26}$$

while $\mathcal{M}_R(t)$ is of the form (13), but with the matrix $\mathcal{R}(t)$ replaced by

$$\mathcal{R}_0(t)=\begin{bmatrix} \cos 2\Omega_0 t+\dfrac{\mathrm{Re}(B)}{2\Omega_0}\sin 2\Omega_0 t & -\dfrac{2\omega+\mathrm{Im}(B)}{2\Omega_0}\sin 2\Omega_0 t \\ \dfrac{2\omega-\mathrm{Im}(B)}{2\Omega_0}\sin 2\Omega_0 t & \cos 2\Omega_0 t-\dfrac{\mathrm{Re}(B)}{2\Omega_0}\sin 2\Omega_0 t \end{bmatrix}, \tag{27}$$

and

$$A=\alpha+a,\quad B\equiv|B|e^{i\phi_B}=\alpha-a+2ib,\quad \Omega_0=\sqrt{\omega^2-|B|^2/4}. \tag{28}$$

The matrix $\mathcal{M}_R(t)$ generalizes that of a rotator. Notice, however, that the oscillator behavior depends on the magnitude of ω with respect to $|B|$; when $\omega<|B|/2$, the frequency Ω_0 becomes purely imaginary and $\mathcal{M}_R(t)$ contains only exponential terms. On the other hand, the form of $\mathcal{M}_D(t)$ looks like the Mueller matrix for a random medium [4]; as in that case, for large times the limit (22) is recovered and any initial polarization is totally lost.

In practical applications, the initial state $|\rho(0)\rangle$ can often be prepared to coincide with that of a linearly polarized photon, given in Eq. (14); one can further set $\theta_0=0$, by a suitable choice of reference frame. Then, inserting the previous results into the general expression (17), the measure of the polarization state along the direction θ after a time t would produce the following intensity pattern:

$$\mathcal{P}_\theta(t)=\frac{1}{2}\left\{1+e^{-At}\left[\cos 2(\theta-\Omega_0 t)+\left(\frac{|B|}{2\Omega_0}\cos(2\theta+\phi_B)+\left(\frac{\omega}{\Omega_0}-1\right)\sin 2\theta\right)\sin 2\Omega_0 t\right]\right\}. \tag{29}$$

As already mentioned, for large times dissipation prevail and all polarization states become equally probable.

The expression in Eq. (29) further simplifies when $\gamma=0$; as observed before, this automatically guarantees $c=\beta=0$ and further imposes $b=0$ and $a=\alpha$. In this case, one explicitly gets

$$\mathcal{P}_\theta(t)=\frac{1}{2}\{1+e^{-2\alpha t}\cos 2(\theta-\omega t)\}. \tag{30}$$

This is the most simple expression that the transition probability $\mathcal{P}_\theta(t)$ takes in presence of dissipative effects.

Another useful approximation of the general formula (17) can be obtained when the nonstandard parameters a, b, c, α, β, and γ are small compared with ω, assumed to be nonvanishing. In this case, the additional piece $\mathcal{L}[\rho]$ in the evolution equation (4) can be treated as a perturbation [15]. All entries in the 3×3 matrix $\mathcal{N}(t)$ are now nonvanishing and in general, the resulting Mueller matrix in Eq. (11) cannot be decomposed as a product of simpler matrices, as in Eq. (25). Explicit expressions for its entries, expanded up to second order in the small parameters, are collected in the Appendix. Using these results, the transition probability $\mathcal{P}_\theta(t)$ takes the form

$$\mathcal{P}_\theta(t)=\frac{1}{2}\left\{1+e^{-At}\left[\cos 2(\theta-\Omega t)+\left(\frac{|B|}{2\Omega}\cos(2\theta+\phi_B)+\left(\frac{\omega}{\Omega}-1\right)\sin 2\theta\right)\sin 2\Omega t+\frac{|C|^2}{\Omega^2}\sin\phi_C\big(\sin(2\theta+\phi_C)\cos 2\Omega t-\sin\phi_C\cos 2\theta\big)\right]\right\}, \tag{31}$$

where A and B are as in Eq. (28), while

$$C \equiv |C| e^{i\phi_C} = c + i\beta, \quad \Omega = \sqrt{\omega^2 - |C|^2 - |B|^2/4}. \quad (32)$$

In writing Eq. (31), we have reconstructed the exponential factors by consistently putting together the terms linear and quadratic in t; a similar treatment has allowed writing the oscillatory contributions in terms of the frequency Ω.

It is worth noting that the expression (31) reduces to the expression in Eq. (29) for $|C|=0$, i.e., when $c=\beta=0$: it is therefore a correction to Eq. (29) for nonvanishing[2] C. In this respect, the validity of Eq. (31) goes beyond the approximation in which it has been derived: it can be considered as the expansion of the full probability $\mathcal{P}_\theta(t)$ up to second order in c and β, and thus it is valid also for vanishing ω.

The behavior of the probabilities in Eqs. (29)–(31), and of other observables that can be similarly constructed, are clearly affected by the presence of dissipation and irreversibility. From the experimental point of view, the actual visibility of such nonstandard effects clearly depends on the magnitude of the parameters a, b, c, α, β, and γ. In a phenomenological approach, it is hard to give an *a priori* estimate on how large the dissipative effects should be. However, as already mentioned in the introductory remarks, a general framework in which dissipation naturally emerges is provided by the study of subsystems in interaction with large environments. In such instances, the nonstandard effects can be roughly estimated to be proportional to the typical energy of the system, while suppressed by inverse powers of the characteristic energy scale of the environment.

In the case of polarized photon beams, these considerations, together with the general idea that dissipation is induced by quantum effects at Planck's scale, lead to predict very small values for the parameters a, b, c, α, β, and γ; for any fixed observational condition, an upper bound on the magnitude of these parameters can be roughly evaluated to be of order E^2/M_P, with E the average photon energy and M_P the Planck mass. This ratio is of order 10^{-49} GeV for a typical radiowave, 10^{-38} GeV for ordinary laboratory laser beams, and 10^{-19} GeV or more for energetic γ-rays.

At first sight, it might look very hard to construct an actual experimental setup sensible to such tiny values. However, the sophistication of present and planned ''optical'' devices is so high [42] that at least some bounds on a, b, c, α, β, and γ should actually be obtainable in the near future. In fact, at least in principle, a very simple setup is needed in order to test the nonstandard dynamics in Eqs. (4), (5). An initially polarized photon beam evolves undisturbed for a time t; its final polarization state is then determined by measuring the corresponding Stokes parameters, and compared with the results in Eq. (11). In practice, it might be more convenient to perform an interferometric polarization test; the resulting intensity curve can be directly compared with the behavior of $\mathcal{P}_\theta(t)$ in Eqs. (29)–(31).

As a simplifying working assumption, take $\gamma=0$, so that only the parameter α survives, as a consequence of Eq. (7); then, from the expression in Eq. (30), the ability of detecting the nonstandard effects is connected to the sensitivity in isolating the exponential factor $e^{-2\alpha t}$ from the experimental data.[3] This can be very high, so that, for long enough t, a good sensitivity on α can be reached.

Although a detailed analysis of possible experimental setups that can be used for the measure of α is surely beyond the scope of the present investigation, some general considerations on the sensitivity of present and future apparatus can be given. In the case of ground-based laboratory experiments using ordinary laser beams, even for relatively large ''storage'' times t, as obtained in high efficient cavities [42,43] or in the interferometric detectors for gravitational waves, [44] the sensitivity on α cannot optimistically exceed 10^{-30} GeV, a few orders of magnitude away from the estimated upper bound on α.

The situation clearly improves using high energetic polarized γ-beams (with $E\sim 10^2$ GeV or greater), as the ones expected in the so-called Photon Colliders, [45] since now one should have $\alpha \leqslant 10^{-15}$ GeV; the required sensitivity for measuring this parameter from Eq. (29) can be reached with a path-length $l\equiv t$ of just a few centimeters. These high-energy accelerators, together with the e^+-e^- linear colliders to which they are coupled, turn out to be particularly suited for studying certain aspects of the standard model [46]; many projects for their actual realization are in advanced stage of development [47]. On the other hand, the construction of polarized photon beams of more modest energies ($E \sim 1$ GeV) is surely in the capacity of any high energy laboratory (e.g., see the discussion presented in [48]). They might actually soon be built with the aim of studying the birefringence effects predicted in [32]; clearly, they would also provide a suitable venue for deriving accurate bounds on some of the parameters a, b, c, α, β, and γ.

Polarized photons of astrophysical and cosmological origin can also be used to probe the presence of the dissipative effects described before. Indeed, radio signals from active galactic nuclei and quasars have already been used to put stringent bounds on *CPT*-violating birefringence effects [35]. For a typical radiowave (with frequency $\sim$1 GHz), the dimensional arguments discussed before would give un upper bound on the magnitude of the dissipative effects that is really very small: $\alpha \leqslant 10^{-49}$ GeV; however, the propagation time t can now be as big as the inverse of the Hubble constant. Therefore, the sensitivity of the present radiotelescope polarization measures is not too far form the above upper limit and improvements can be expected in the future [49].

[2]Analogously, in the same limit the elements of $\mathcal{M}(t)$ listed in the Appendix reduces to those in Eqs. (25)–(27).

[3]As already mentioned, also ω contains in general dissipative contributions, resulting from the interaction with the environment. However, these contributions to birefringence cannot be disentangled from those produced by other physical effects (see [32–38]). On the other hand, the dependence of $\mathcal{P}_\theta$ on the nonstandard parameters a, b, c, α, β, and γ is distinctive of dissipative phenomena and cannot be mimicked by other unconventional mechanisms.

The development of very sophisticated detectors, both ground-based and space-based, has allowed the observation and the study of γ-ray emissions from a variety of astrophysical sources [50,51]. In view of their extremely high energy (ranging from 10^2 KeV up to 1 TeV or more) and of their extragalactic or cosmological origin (resulting in large propagation times t), these photons turn out to be a very interesting system for measuring nonstandard, dissipative effects. In fact, the physical mechanisms that have been proposed to explain the origin of these energetic emissions give rise to (partially) polarized photons [50–52], and preliminary observations seem to confirm this prediction [53,54]. If efficient polarimeters will be coupled to the next generation of orbiting γ-ray spectrometers, accurate measurements of the parameters in Eq. (10) might indeed be possible.

In conclusion, the study of polarized photon beams can provide very useful information on the presence of dissipation and irreversibility induced by a fundamental ''stringy'' dynamics. Future experiments, in the laboratory and in space, will likely be able to put stringent bounds on these nonstandard effects.

APPENDIX

As discussed in the text, when the parameters a, b, c, α, β, and γ can be considered small with respect to ω, perturbation theory can be used to find a convenient explicit expression for the solution of Eq. (9). Up to second order in the small parameters, the entries of the matrix $\mathcal{N}(t)$ in Eq. (11) can then be written as

$$\mathcal{N}_{11}(t)=e^{-At}\left[\cos 2\Omega t+\frac{\mathrm{Re}(B)}{2\Omega}\sin 2\Omega t\right] -2\left[\frac{\mathrm{Im}(C)}{\Omega}\right]^2\sin^2\Omega t,$$

$$\mathcal{N}_{12}(t)=-e^{-At}\left[\frac{2\omega+\mathrm{Im}(B)}{2\Omega}\right]\sin 2\Omega t+\frac{\mathrm{Im}(C^2)}{2\Omega^2}\cos 2\Omega t,$$

$$\mathcal{N}_{13}(t)=\frac{\mathrm{Im}(C)}{\Omega}e^{-2\gamma t}-e^{-At}\frac{|C|}{\Omega}\sin(2\Omega t+\phi_C)-[|C|(A-2\gamma)\sin(\Omega t+\phi_C)+\mathrm{Re}(BC)\sin\Omega t]\frac{\sin\Omega t}{\Omega^2},$$

$$\mathcal{N}_{22}(t)=e^{-At}\left[\cos 2\Omega t-\frac{\mathrm{Re}(B)}{2\Omega}\sin 2\Omega t\right]-2\left[\frac{\mathrm{Re}(C)}{\Omega}\right]^2\sin^2\Omega t,$$

$$\mathcal{N}_{23}(t)=-\frac{\mathrm{Re}(C)}{\Omega}e^{-2\gamma t}+e^{-At}\frac{|C|}{\Omega}\cos(2\Omega t+\phi_C)+[|C|(A-2\gamma)\cos(\Omega t+\phi_C)+\mathrm{Im}(BC)\sin\Omega t]\frac{\sin\Omega t}{\Omega^2},$$

$$\mathcal{N}_{33}(t)=e^{-2\gamma t}+\frac{2|C|^2}{\Omega^2}\sin^2\Omega t,$$

where the definitions (28) and (32) have been used; the remaining off-diagonal elements $\mathcal{N}_{21}$, $\mathcal{N}_{31}$ and $\mathcal{N}_{32}$ can be obtained from $\mathcal{N}_{12}$, $\mathcal{N}_{13}$ and $\mathcal{N}_{23}$, respectively, by letting $\omega\to-\omega$ and $\Omega\to-\Omega$.

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